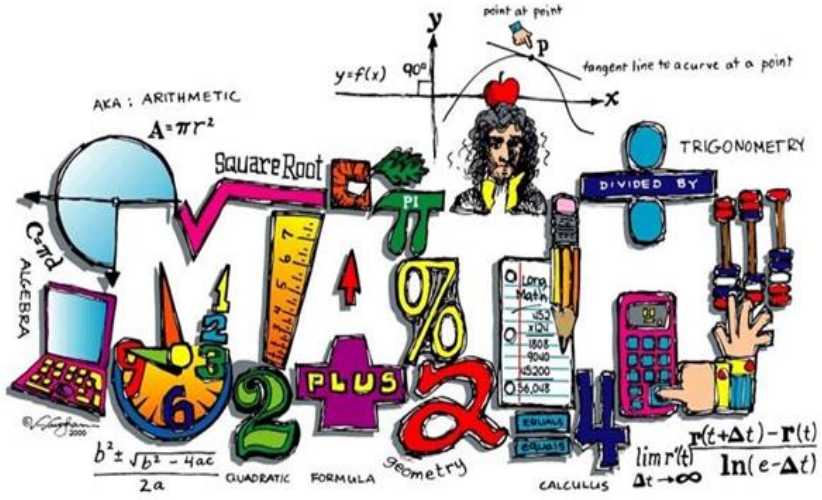
MATHEMATICS



Algebra II: Unit 1

Complex Solutions and Modeling with Rational Exponent

Course Philosophy/Description

Algebra II continues the students' study of advanced algebraic concepts including functions, polynomials, rational expressions, systems of functions and inequalities, and matrices. Students will be expected to describe and translate among graphic, algebraic, numeric, tabular, and verbal representations of relations and use those representations to solve problems. Emphasis will be placed on practical applications and modeling. Students extend their knowledge and understanding by solving open-ended real-world problems and thinking critically through the use of high level tasks.

Students will be expected to demonstrate their knowledge in: utilizing essential algebraic concepts to perform calculations on polynomial expression; performing operations with complex numbers and graphing complex numbers; solving and graphing linear equations/inequalities and systems of linear equations/inequalities; solving, graphing, and interpreting the solutions of quadratic functions; solving, graphing, and analyzing solutions of polynomial functions, including complex solutions; manipulating rational expressions, solving rational equations, and graphing rational functions; solving logarithmic and exponential equations; and performing operations on matrices and solving matrix equations.

ESL Framework

This ESL framework was designed to be used by bilingual, dual language, ESL and general education teachers. Bilingual and dual language programs use the home language and a second language for instruction. ESL teachers and general education or bilingual teachers may use this document to collaborate on unit and lesson planning to decide who will address certain components of the SLO and language objective. ESL teachers may use the appropriate leveled language objective to build lessons for ELLs which reflects what is covered in the general education program. In this way, whether it is a pull-out or push-in model, all teachers are working on the same Student Learning Objective connected to the New Jersey Student Learning Standards. The design of language objectives are based on the alignment of the World-Class Instructional Design Assessment (WIDA) Consortium's English Language Development (ELD) standards with the New Jersey Student Learning Standards (NJSLS). WIDA's ELD standards advance academic language development across content areas ultimately leading to academic achievement for English learners. As English learners are progressing through the six developmental linguistic stages, this framework will assist all teachers who work with English learners to appropriately identify the language needed to meet the requirements of the content standard. At the same time, the language objectives recognize the cognitive demand required to complete educational tasks. Even though listening and reading (receptive) skills differ from speaking and writing (expressive) skills across proficiency levels the cognitive function should not be diminished. For example, an Entering Level One student only has the linguistic ability to respond in single words in English with significant support from their home language. However, they could complete a Venn diagram with single words which demonstrates that they understand how the elements compare and contrast with each other or they could respond with the support of their home language (L1) with assistance from a teacher, para-professional, peer or a technology program.

STATE OF NEW JERSEY
DEPARTMENT OF EDUCATION

http://www.state.nj.us/education/modelcurriculum/ela/ELLOverview.pdf

Pacing Chart – Unit 1

#	Student Learning Objective	NJSLS	
1	Add, subtract, and multiply complex numbers using the commutative, associative and distributive properties.	N.CN.A.1. N.CN.A.2.	**Instruction: 8 weeks** **Assessment: 1 week**
2	Solve quadratic equations with real coefficients that have complex solutions by taking square roots, completing the square and factoring.	N.CN.C.7. A.REI.B.4	
3	Solve simple systems consisting of a linear and quadratic equation in two variables algebraically and graphically.	A.REI.C.7	
4	Solve algebraically a system of three linear equations.	A.REI.C.6	
5	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.	F.BF.A.2. F.LE.A.2 F.LE.B.5.	
6	Use the formula for the sum of a finite geometric series to solve problems [for example, calculate mortgage payments; derive the formula for the sum of a finite geometric series (when the common ratio is not 1)].	A.SSE.B.4.	
7	Use properties of integer exponents to explain and convert between expressions involving radicals and rational exponents.	N.RN.A.1. N.RN.A.2	
8	Use the properties of exponents to transform expressions for exponential functions, explain properties of the quantity revealed in the transformed expression or different properties of the function.	A.SSE.B.3 F.IF.C.8.	
9	Express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.	F.LE.A.4.	

Research about Teaching and Learning Mathematics

Structure teaching of mathematical concepts and skills around problems to be solved (Checkly, 1997; Wood & Sellars, 1996; Wood & Sellars, 1997)
Encourage students to work cooperatively with others (Johnson & Johnson, 1975; Davidson, 1990)
Use group problem-solving to stimulate students to apply their mathematical thinking skills (Artzt & Armour-Thomas, 1992)
Students interact in ways that support and challenge one another's strategic thinking (Artzt, Armour-Thomas, & Curcio, 2008)
Activities structured in ways allowing students to explore, explain, extend, and evaluate their progress (National Research Council, 1999)
There are three critical components to effective mathematics instruction (Shellard & Moyer, 2002):

- Teaching for conceptual understanding
- Developing children's procedural literacy
- Promoting strategic competence through meaningful problem-solving investigations

Teachers should be:

- Demonstrating acceptance and recognition of students' divergent ideas.
- Challenging students to think deeply about the problems they are solving, extending thinking beyond the solutions and algorithms required to solve the problem
- Influencing learning by asking challenging and interesting questions to accelerate students' innate inquisitiveness and foster them to examine concepts further.
- Projecting a positive attitude about mathematics and about students' ability to "do" mathematics

Students should be:

- Actively engaging in "doing" mathematics
- Solving challenging problems
- Investigating meaningful real-world problems
- Making interdisciplinary connections
- Developing an understanding of mathematical knowledge required to "do" mathematics and connect the language of mathematical ideas with numerical representations
- Sharing mathematical ideas, discussing mathematics with one another, refining and critiquing each other's ideas and understandings
- Communicating in pairs, small group, or whole group presentations
- Using multiple representations to communicate mathematical ideas
- Using connections between pictures, oral language, written symbols, manipulative models, and real-world situations
- Using technological resources and other 21st century skills to support and enhance mathematical understanding

Mathematics is not a stagnate field of textbook problems; rather, it is a dynamic way of constructing meaning about the world around us, generating knowledge and understanding about the real world every day. Students should be metaphorically rolling up their sleeves and "doing mathematics" themselves, not watching others do mathematics for them or in front of them. (Protheroe, 2007)

Conceptual-Based Model

The purpose of the Conceptual-Based Model is to allow students the time to explore mathematical concepts to promote academic rigor and high level of student discourse to concurrently develop conceptual understanding, procedural fluency, and problem-solving skills. During the Mathematical block of instruction, teachers will select and set up a mathematical task that targets the mathematical goal(s) for the lesson. The teacher sets the stage for learning by ensuring the objective/rationale of the lesson is well-defined and connected to the task. The task should build on student's prior knowledge, life experiences, and culture allowing students to share their prior knowledge and life/cultural experiences as it relates to the task to ensure that students understand the context of the problem. The instructional goal is to introduce the activity/task to the students allowing them to have access to learning while maintaining the cognitive demands of the task. Teachers will then support the students' exploration of the task; this can be done independently, in pairs or in small groups or a combination of all. It is highly recommended that students be given the opportunity to privately work on a task to generate solutions on their own. Students are encouraged to share their findings with their peers in small group to compare their solutions. As students are actively engaged in constructing meaning of the mathematical concept(s) being taught and communicating their understanding of the concept(s) with their peers, the teacher monitors the development of student understanding by observing student thinking and using questions to stimulate thinking to drive students toward the aimed mathematical goal(s). The teacher assesses students' understanding of key mathematical ideas, problem-solving strategies, and the use of and connection between models and representations to determine what the student knows. The teacher advances the students' understanding to move the student beyond their present thinking and expand what they know to an additional situation. Teachers have been trained to strategically select groups of students who have different solution paths to the same task, different representations and errors/misconceptions to share, discuss, and analyze as a whole group. By providing these instructional opportunities, the teacher will then be able to orchestrate the class discussion by providing students with the opportunities to make their learning public as students share, discuss, analyze, clarify, extend, connect, strengthen, and record their thinking strategies. After students discuss, justify, and challenge the various solution paths that were shared, a summary of the learning is articulated and connected to the objective of the lesson. Students should be given an opportunity to close the lesson with a reflection on their learning.

Effective Pedagogical Routines/Instructional Strategies	
Collaborative Problem Solving	Analyze Student Work
Connect Previous Knowledge to New Learning	Identify Student's Mathematical Understanding
Making Thinking Visible	Identify Student's Mathematical Misunderstandings
Develop and Demonstrate Mathematical Practices	Interviews
Inquiry-Oriented and Exploratory Approach	Role Playing
Multiple Solution Paths and Strategies	Diagrams, Charts, Tables, and Graphs
Use of Multiple Representations	Anticipate Likely and Possible Student Responses
Explain the Rationale of your Math Work	Collect Different Student Approaches
Quick Writes	Multiple Response Strategies
Pair/Trio Sharing	Asking Assessing and Advancing Questions
Turn and Talk	Revoicing
Charting	Marking
Gallery Walks	Recapping
Small Group and Whole Class Discussions	Challenging
Student Modeling	Pressing for Accuracy and Reasoning
	Maintain the Cognitive Demand

Educational Technology

Standards

8.1.12.A.1, 8.1.12.A.5, 8.1.12.D.1, 8.1.12.E.1, 8.2.12.B.1

- **Technology Operations and Concepts**
  - Create professional documents (e.g., newsletter, personalized learning plan, business letter or flyer) using advanced features of a word processing program.
  - Select and use appropriate tools and digital resources to accomplish a variety of tasks and to solve problems.
- **Digital Citizenship**
  - Model appropriate online behaviors related to cyber safety, cyber bullying, cyber security, and cyber ethics.
- **Research and Information Literacy**
  - Gather and analyze findings to produce a possible solution for a content-related or real world problem using data collection technology.
- **Design: Critical Thinking, Problem Solving, and Decision Making**
  - Design and create a product using the design process that addresses a real world problem with specific criteria and constraints.

Career Ready Practices

Career Ready Practices describe the career-ready skills that all educators in all content areas should seek to develop in their students. They are practices that have been linked to increase college, career, and life success. Career Ready Practices should be taught and reinforced in all career exploration and preparation programs with increasingly higher levels of complexity and expectation as a student advances through a program of study**.**

- **CRP1. Act as a responsible and contributing citizen and employee**
  Career-ready individuals understand the obligations and responsibilities of being a member of a community, and they demonstrate this understanding every day through their interactions with others. They are conscientious of the impacts of their decisions on others and the environment around them. They think about the near-term and long-term consequences of their actions and seek to act in ways that contribute to the betterment of their teams, families, community and workplace. They are reliable and consistent in going beyond the minimum expectation and in participating in activities that serve the greater good.

- **CRP2. Apply appropriate academic and technical skills.**
  Career-ready individuals readily access and use the knowledge and skills acquired through experience and education to be more productive. They make connections between abstract concepts with real-world applications, and they make correct insights about when it is appropriate to apply the use of an academic skill in a workplace situation

- **CRP3. Attend to personal health and financial well-being.**
  Career-ready individuals understand the relationship between personal health, workplace performance and personal well-being; they act on that understanding to regularly practice healthy diet, exercise and mental health activities. Career-ready individuals also take regular action to contribute to their personal financial wellbeing, understanding that personal financial security provides the peace of mind required to contribute more fully to their own career success.

- **CRP4. Communicate clearly and effectively and with reason.**
  Career-ready individuals communicate thoughts, ideas, and action plans with clarity, whether using written, verbal, and/or visual methods. They communicate in the workplace with clarity and purpose to make maximum use of their own and others' time. They are excellent writers; they master conventions, word choice, and organization, and use effective tone and presentation skills to articulate ideas. They are skilled at interacting with others; they are active listeners and speak clearly and with purpose. Career-ready individuals think about the audience for their communication and prepare accordingly to ensure the desired outcome.

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- **CRP5. Consider the environmental, social and economic impacts of decisions.**
  Career-ready individuals understand the interrelated nature of their actions and regularly make decisions that positively impact and/or mitigate negative impact on other people, organization, and the environment. They are aware of and utilize new technologies, understandings, procedures, materials, and regulations affecting the nature of their work as it relates to the impact on the social condition, the environment and the profitability of the organization.

- **CRP6. Demonstrate creativity and innovation.**
  Career-ready individuals regularly think of ideas that solve problems in new and different ways, and they contribute those ideas in a useful and productive manner to improve their organization. They can consider unconventional ideas and suggestions as solutions to issues, tasks or problems, and they discern which ideas and suggestions will add greatest value. They seek new methods, practices, and ideas from a variety of sources and seek to apply those ideas to their own workplace. They take action on their ideas and understand how to bring innovation to an organization.

- **CRP7. Employ valid and reliable research strategies.**
  Career-ready individuals are discerning in accepting and using new information to make decisions, change practices or inform strategies. They use reliable research process to search for new information. They evaluate the validity of sources when considering the use and adoption of external information or practices in their workplace situation.

- **CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.**
  Career-ready individuals readily recognize problems in the workplace, understand the nature of the problem, and devise effective plans to solve the problem. They are aware of problems when they occur and take action quickly to address the problem; they thoughtfully investigate the root cause of the problem prior to introducing solutions. They carefully consider the options to solve the problem. Once a solution is agreed

Career Ready Practices

upon, they follow through to ensure the problem is solved, whether through their own actions or the actions of others.

- **CRP10. Plan education and career paths aligned to personal goals**.
  Career-ready individuals take personal ownership of their own education and career goals, and they regularly act on a plan to attain these goals. They understand their own career interests, preferences, goals, and requirements. They have perspective regarding the pathways available to them and the time, effort, experience and other requirements to pursue each, including a path of entrepreneurship. They recognize the value of each step in the education and experiential process, and they recognize that nearly all career paths require ongoing education and experience. They seek counselors, mentors, and other experts to assist in the planning and execution of career and personal goals.

- **CRP11. Use technology to enhance productivity.**
  Career-ready individuals find and maximize the productive value of existing and new technology to accomplish workplace tasks and solve workplace problems. They are flexible and adaptive in acquiring new technology. They are proficient with ubiquitous technology applications. They understand the inherent risks-personal and organizational-of technology applications, and they take actions to prevent or mitigate these risks.

WIDA Proficiency Levels

At the given level of English language proficiency, English language learners will process, understand, produce or use

6- Reaching	• Specialized or technical language reflective of the content areas at grade level • A variety of sentence lengths of varying linguistic complexity in extended oral or written discourse as required by the specified grade level • Oral or written communication in English comparable to proficient English peers
5- Bridging	• Specialized or technical language of the content areas • A variety of sentence lengths of varying linguistic complexity in extended oral or written discourse, including stories, essays or reports • Oral or written language approaching comparability to that of proficient English peers when presented with grade level material.
4- Expanding	• Specific and some technical language of the content areas • A variety of sentence lengths of varying linguistic complexity in oral discourse or multiple, related sentences or paragraphs • Oral or written language with minimal phonological, syntactic or semantic errors that may impede the communication, but retain much of its meaning, when presented with oral or written connected discourse, with sensory, graphic or interactive support
3- Developing	• General and some specific language of the content areas • Expanded sentences in oral interaction or written paragraphs • Oral or written language with phonological, syntactic or semantic errors that may impede the communication, but retain much of its meaning, when presented with oral or written, narrative or expository descriptions with sensory, graphic or interactive support
2- Beginning	• General language related to the content area • Phrases or short sentences • Oral or written language with phonological, syntactic, or semantic errors that often impede of the communication when presented with one to multiple-step commands, directions, or a series of statements with sensory, graphic or interactive support
1- Entering	• Pictorial or graphic representation of the language of the content areas • Words, phrases or chunks of language when presented with one-step commands directions, WH-, choice or yes/no questions, or statements with sensory, graphic or interactive support

Differentiated Instruction

Accommodate Based on Students Individual Needs: Strategies

Time/General	Processing	Comprehension	Recall
• Extra time for assigned tasks • Adjust length of assignment • Timeline with due dates for reports and projects • Communication system between home and school • Provide lecture notes/outline	• Extra Response time • Have students verbalize steps • Repeat, clarify or reword directions • Mini-breaks between tasks • Provide a warning for transitions • Partnering	• Precise processes for conceptual model • Short manageable tasks • Brief and concrete directions • Provide immediate feedback • Small group instruction • Emphasize multi-sensory learning	• Teacher-made checklist • Use visual graphic organizers • Reference resources to promote independence • Visual and verbal reminders • Graphic organizers
Assistive Technology	**Tests/Quizzes/Grading**	**Behavior/Attention**	**Organization**
• Computer/whiteboard • Tape recorder • Video Tape	• Extended time • Study guides • Shortened tests • Read directions aloud	• Consistent daily structured routine • Simple and clear classroom rules • Frequent feedback	• Individual daily planner • Display a written agenda • Note-taking assistance • Color code materials

Interdisciplinary Connections

Model interdisciplinary thinking to expose students to other disciplines.

Social Studies Connection:

Name of Task: Carbon 14 Dating

- The task requires the student to use logarithms to solve an exponential equation in the realistic context of carbon dating, important in archaeology and geology, among other places

Science Connection:

Name of Task: Course of Antibiotics

- This task presents a real world application of finite geometric series. The context can lead into several interesting follow-up questions and projects. Many drugs only become effective after the amount in the body builds up to a certain level. This can be modeled very well with geometric series.

Enrichment

What is the Purpose of Enrichment?

- The purpose of enrichment is to provide extended learning opportunities and challenges to students who have already mastered, or can quickly master, the basic curriculum. Enrichment gives the student more time to study concepts with greater depth, breadth, and complexity.
- Enrichment also provides opportunities for students to pursue learning in their own areas of interest and strengths.
- Enrichment keeps advanced students engaged and supports their accelerated academic needs.
- Enrichment provides the most appropriate answer to the question, "What do you do when the student already knows it?"

Enrichment is…

- Planned and purposeful
- *Different*, or differentiated, work – not just *more* work
- Responsive to students' needs and situations
- A promotion of high-level thinking skills and making connections within content
- The ability to apply different or multiple strategies to the content
- The ability to synthesize concepts and make real world and cross-curricular connections.
- Elevated contextual complexity
- Sometimes independent activities, sometimes direct instruction
- Inquiry based or open ended assignments and projects
- Using supplementary materials in addition to the normal range of resources.
- Choices for students
- Tiered/Multi-level activities with Flexible groups (may change daily or weekly)

Enrichment is not…

- Just for gifted students (some gifted students may need intervention in some areas just as some other students may need frequent enrichment)
- Worksheets that are more of the same (busywork)
- Random assignments, games, or puzzles not connected to the content areas or areas of student interest
- Extra homework
- A package that is the same for everyone
- Thinking skills taught in isolation
- Unstructured free time

Assessments

Required District/State Assessments

Unit 1 Assessment
PARCC
SGO Baseline Assessment
Star Math

Suggested Formative/Summative Classroom Assessments

Describe Learning Vertically
Identify Key Building Blocks
Make Connections (between and among key building blocks)
Short/Extended Constructed Response Items
Multiple-Choice Items (where multiple answer choices may be correct)
Drag and Drop Items
Use of Equation Editor
Quizzes
Journal Entries/Reflections/Quick-Writes
Accountable talk
Projects
Portfolio
Observation
Graphic Organizers/ Concept Mapping
Presentations
Role Playing
Teacher-Student and Student-Student Conferencing
Homework

New Jersey Student Learning Standards

N.CN.A.1.
Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.

N.CN.A.2.
Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers

N.CN.C.7.
Solve quadratic equations with real coefficients that have complex solutions.

A.REI.B.4.
Solve quadratic equations in one variable.

A.REI.B.4b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

A.REI.C.7.
Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line* $y = -3x$ and the circle $x^2 + y^2 = 3$.

A.REI.C.6.
Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

F.BF.A.2.
Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

F.LE.A.2
Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

New Jersey Student Learning Standards

F.LE.B.5.
Interpret the parameters in a linear or exponential function in terms of a context.

A.SSE.B.4.
Derive and/or explain the derivation of the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*

N.RN.A.1.
Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define* $5^{1/3}$ *to be the cube root of 5 because we want* $(5^{1/3})^3 = 5(^{1/3})^3$ *to hold, so* $(5^{1/3})^3$ *must equal 5.*

N.RN.A.2.
Rewrite expressions involving radicals and rational exponents using the properties of exponents.

A.SSE.B.3.
Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression

A.SSE.B.3c: Use the properties of exponents to transform expressions for exponential functions. *For example the expression* 1.15^t *can be rewritten as* $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*

F.IF.C.8.
Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function

F.IF.C.8b: Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, *and classify them as representing exponential growth or decay.*

F.LE.A.4.
Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

Mathematical Practices

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with mathematics.**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**

Grade: Algebra II	Unit: 1 (One)	Topic: Complex Solutions and Modeling with Rational Exponents

NJSLS:

N.CN.A.1, N.CN.A.2, N.CN.C.7, A.REI.B.4, A.REI.B.4b, A.REI.C.7, A.REI.C.6, F.BF.A.2, F.LE.A.2, F.LE.B.5, A.SSE.B.4, N.RN.A.1. N.RN.A.2, A.SSE.B.3, F.IF.C.8, F.IF.C.8b, F.LE.A.4

Unit Focus:

- **Perform arithmetic operations with complex numbers**
- **Use complex numbers in polynomial identities and equations**
- **Build a function that models a relationship between two quantities**
- **Construct & compare linear, quadratic, & exponential models**
- **Write expressions in equivalent forms to solve problems**
- **Extend the properties of exponents to rational exponents**
- **Analyze functions using different representations**

New Jersey Student Learning Standard(s):

N.CN.A.1: Know there is a complex number i such that $i^2 = –1$, and every complex number has the form $a + bi$ with a and b real.

N.CN.A.2: Use the relation $i^2 = –1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers

Student Learning Objective 1: Add, subtract, and multiply complex numbers using the commutative, associative and distributive properties.

Modified Student Learning Objectives/Standards:

M.EE.N-CN.2.a. Use the commutative, associative, and distributive properties to add, subtract, and multiply whole numbers.
M.EE.N-CN.2.b. Solve real-world problems involving addition and subtraction of decimals, using models when needed.
M.EE.N-CN.2.c. Solve real-world problems involving multiplication of decimals and whole numbers, using models when needed.

MPs	Evidence Statement Key/ Clarifications	Skills, Strategies & Concepts	Essential Understandings/ Questions (Accountable Talk)	Tasks/Activities
MP 6	**N-CN.1**	Complex number i is defined such that $i^2 = –1$.	What are the subsets of the set of complex numbers?	**Type I: Computations with**

MP 7	**N-CN.2**	Every complex number has the form $a + bi$ with a and b real. $i^2 = –1$ and the commutative, associative properties to add and subtract complex numbers are to be used. Determine that $i^2 = –1$ and the commutative, associative, and distributive properties to multiply complex numbers. **SPED Strategies:** Relate the idea of adding, subtracting and multiplying complex numbers to whole numbers**.** Explain the background of complex numbers and connect to real life by explaining how they are used in electrical circuits. Use the example of the cyclical nature of the ones digit in the powers of 3 and connect it to the cyclical nature of the powers of i. Develop a reference document with students with verbal and pictorial descriptions.	How are the powers of i derived and how are they cyclic? Why is it when simplifying imaginary and complex numbers that the highest power of i is one?	**Complex Numbers** **Product of the complex numbers** **Type II, III:** **Complex number patterns** **Appropriate cells** **Powers of a complex number** **Complex Square Roots** **Additional Tasks: Complex Cube and Fourth Root of 1** **Vertex of a parabola with complex roots**

New Jersey Student Learning Standard(s):
N.CN.C.7: Solve quadratic equations with real coefficients that have complex solutions.

A.REI.B.4: Solve quadratic equations in one variable.

A.REI.B.4b: Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

Student Learning Objective 2: Solve quadratic equations with real coefficients that have complex solutions by taking square roots, completing the square and factoring.

Modified Student Learning Objectives/Standards: N/A

MPs	Evidence Statement Key/ Clarifications	Skills, Strategies & Concepts	Essential Understandings/ Questions (Accountable Talk)	Tasks/Activities
MP 5 **MP 7**	N.CN.C.7 • Tasks are limited to equations with non-real solutions. **A-REI.4b-2** • Writing solutions in the form $a \pm bi$ is not assessed here (assessed under N-CN.7).	As with real solutions, complex solutions to quadratic equations may be determined by taking square roots, factoring, and completing the square. Evaluate powers of í. Solve quadratic equations with complex solutions. Solve quadratic equations using the square root method. Solve quadratic equations by factoring and using the zero product property. Solve quadratic equations in one variable that have complex solutions by taking square roots.	What are the subsets of the set of complex numbers? How can you complete the square for a quadratic expression? How can you derive a general formula for solving a quadratic equation? How can you determine whether a polynomial equation has imaginary solutions? How do you determine which method is best for solving a quadratic equation?	**Type II, III: Two Squares are Equal** **Braking Distance** **Finding the m** **Additional Tasks: Completing the square** **Springboard Dive** **Vertex of a parabola with complex roots**

		Solve a quadratic equation in one variable that have complex solutions by completing the square. Solve quadratic equations in one variable that have complex solutions by factoring. Write complex solutions in $a \pm bi$ form. **SPED Strategies** Model the thinking and procedure involved in solving a quadratic equation with complex solutions. Provide students with a graphic organizer that outlines the possible solution paths, formulas and sample problems to facilitate independence. Encourage students to verbalize their thinking while solving quadratic equations by asking assessing and advancing questions.	Why do some quadratic equations have extraneous and/or complex solutions? Why is it when simplifying imaginary and complex numbers that the highest power of í is one? How does the concept of the zero product property allow you to find the roots of a quadratic function?	**Visualizing completing the square** **Zero Property 4**

New Jersey Student Learning Standard(s):
A.REI.C.7: Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line* $y = -3x$ and the circle $x^2 + y^2 = 3$.

Student Learning Objective 3: Solve simple systems consisting of a linear and quadratic equation in two variables algebraically and graphically.

Modified Student Learning Objectives/Standards:
M.EE.A-REI.10–12: Interpret the meaning of a point on the graph of a line. For example, on a graph of pizza purchases, trace the graph to a point and tell the number of pizzas purchased and the total cost of the pizzas.

MPs	Evidence Statement Key/ Clarifications	Skills, Strategies & Concepts	Essential Understandings/ Questions (Accountable Talk)	Tasks/Activities
MP 1	**A.REI.C.7** • Tasks have thin context or no context.	Solutions of linear systems contain different function types. Solve a system containing one linear equation and one quadratic equation algebraically. Graph a system containing one linear equation and one quadratic equation to determine a solution. **SPED Strategies** Link the concept of solving a system of equations with one linear and one quadratic equation to solving a system of linear equations. Model the thinking and processes necessary to decide on a solution path and solve a system with one linear equation and one quadratic equation accurately.	How can you solve a nonlinear system of equations? How can you solve a system of two equations when one is linear and the other is quadratic? How do you determine the number of solutions that a system of equations will have? Why does graphing a system of equation yield an approximate solution as opposed to an exact solution?	**Type II, III: The Circle and The Line** **Pythagorean Triples** **Flying T-shirts** **Linear and Quadratic**

		Provide students with reference sheets/notes to encourage confidence and independence.		

New Jersey Student Learning Standard(s):
A.REI.C.6: Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Student Learning Objective 4: Solve algebraically a system of three linear equations.

Modified Student Learning Objectives/Standards: See EE.A-REI.10–12.
EE.A-REI.10–12. Interpret the meaning of a point on the graph of a line. For example, on a graph of pizza purchases, trace the graph to a point and tell the number of pizzas purchased and the total cost of the pizzas.

MPs	Evidence Statement Key/ Clarifications	Skills, Strategies & Concepts	Essential Understandings/ Questions (Accountable Talk)	Tasks/Activities
MP 1 **MP 7**	**A.REI.C.6** • Coefficients are rational numbers. • Tasks do not require any specific method to be used (e.g., prompts do not direct the student to use elimination or any other particular method).	Solving a system of linear equations containing *n* variables requires *n* equations. Use the substitution method and/or elimination method to find the solution of a system containing three linear equations. **SPED Strategies:** Model the thinking and processes necessary to decide on a solution path and solve a system of three linear equations accurately. Provide students with reference sheets/notes to encourage confidence and independence.	How can you determine the number of solutions of a linear system? How can you use substitution to solve a system of linear equations? How can you use elimination to solve a system of linear equations? Can a system of equations have no solution or infinitely many solutions? How do you determine the best	**Type II, III:** **Cash Box** **Pairs of Whole Numbers** **Estimating a Solution via Graphs** **Find A System**

			method for solving a given system of equation?	

New Jersey Student Learning Standard(s):

F.BF.A.2: Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

F.LE.A.2: Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

F.LE.B.5: Interpret the parameters in a linear or exponential function in terms of a context.

Student Learning Objective 5:

Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

Modified Student Learning Objectives/Standards:

M.EE.F-BF.2: Determine an arithmetic sequence with whole numbers when provided a recursive rule.

M.EE.F-LE.1–3: Model a simple linear function such as y = mx to show that these functions increase by equal amounts over equal intervals.

MPs	Evidence Statement Key/ Clarifications	Skills, Strategies & Concepts	Essential Understandings/ Questions (Accountable Talk)	Tasks/Activities
MP 1 **MP 2** **MP 4** **MP 6** **MP 7** **MP 8**	**F-BF.2** **F-LE.2-3** • Solve multi-step contextual problems with degree of difficulty appropriate to the course by constructing linear and/or exponential function models. • Prompts describe a	Distinguish between recursive and explicit formulas. Represent geometric and arithmetic sequences recursively. Represent geometric and arithmetic sequences with explicit formulas. Translate between recursive form and explicit form of geometric and arithmetic sequences.	How can you recognize an arithmetic sequence from its graph? How can you recognize a geometric sequence from its graph? How can you define a sequence recursively? What are some of the	**Type II, III:** **Decaying Dice** **Rumors** **Taxi!** **Saving** **Additional Tasks:** **Exponential**

	scenario using everyday language. Mathematical language such as "function," "exponential," etc. is not used. • Students autonomously choose and apply appropriate mathematical techniques without prompting. For example, in a situation of doubling, they apply techniques of exponential functions. • For some illustrations, see tasks at http://illustrativemathematics.org under F-LE.	Recognize explicit formula for geometric sequences as exponential functions containing a domain in the integers only. Interpret the parameters of an exponential function representing a geometric sequence. Interpret the parameters of a linear function representing an arithmetic sequence. **SPED Strategies:** Review the differences between geometric and arithmetic sequences by giving students examples and illustrating the characteristics that distinguish them. Pre-teach the vocabulary and provide verbal and pictorial descriptions. (i.e. recursive and explicit formulas). Model the thinking and procedure involved in writing geometric and arithmetic sequences in recursive and explicit form. Provide students with a graphic organizer/reference sheet/Google Doc that highlights the thinking and procedure involved in writing geometric and arithmetic sequences in recursive and explicit form.	characteristics of the graph of an exponential function? How can you recognize polynomial, exponential, and logarithmic models? What data do you need to write a function to model a given situation? How do you determine if a given situation is modeled by a linear or exponential function? Create an example of a linear or exponential situation and give the function that can be used to model the situation. What does each part of the function represent in the context of the problem and what are the parameters?	**Parameters** **Kimi and Jordan** **Population and Food Supply** **Snake on a Plane** **Susita's Account** **Canoe Trip variation 1** **Canoe Trip variation 2** **US Population 1982-1988**

New Jersey Student Learning Standard(s):
A.SSE.B.4: Derive and/or explain the derivation of the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*

Student Learning Objective 6: Use the formula for the sum of a finite geometric series to solve problems [*for example, calculate the mortgage payments*; derive the formula for the sum of a finite geometric series (when the common ratio is not 1)].

Modified Student Learning Objectives/Standards:
M.EE.A-SSE.4. Determine the successive term in a geometric sequence given the common ratio.

MPs	Evidence Statement Key/ Clarifications	Skills, Strategies & Concepts	Essential Understandings/ Questions (Accountable Talk)	Tasks/Activities
MP 1 **MP 7**	**A-SSE.4-2** • Use the formula for the sum of a finite geometric series to solve multi-step contextual problems. • In a multistep task, students may be expected to calculate the value of a single term as well as the sum.	Series as a sum of a sequence. Derive or explain the derivation of the formula for the sum of a finite geometric series. Use the formula for the sum of a finite geometric series to solve problems. **SPED Strategies:** Pre-teach vocabulary and provide verbal and pictorial descriptions to maximize understanding and interest. Introduce the concept imbedded in a real-life context to help students relate to and internalize the mathematics involved. Model the thinking and processes involved in	How can students recognize a geometric sequence from its graph? How can you find the sum of an infinite geometric series?	**Type II, III:** **Course of Antibiotics** **YouTube Explosion** **Additional Tasks:** **Triangle Series** **A Lifetime of Savings** **Cantor Set**

		solving problems involving finite geometric series. Provide students with a graphic organizer/reference sheet/Google Doc that highlights the thinking and procedure involved solving problems involving finite geometric series.		

New Jersey Student Learning Standard(s):

N.RN.A.1: Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5(^{1/3})^3$ to hold, so $(5^{1/3})^3$ must equal 5.*

N.RN.A.2: Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Student Learning Objective 7: Use properties of integer exponents to explain and convert between expressions involving radicals and rational exponents.

Modified Student Learning Objectives/Standards:

M.EE.N-RN.1: Determine the value of a quantity that is squared or cubed.

MPs	Evidence Statement Key/ Clarifications	Skills, Strategies & Concepts	Essential Understandings/ Questions (Accountable Talk)	Tasks/Activities
MP 7	**N-RN.2**	Properties of integer exponents extends to rational exponents (*for example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5(^{1/3})^3$ to hold, so $(5^{1/3})^3$ must equal 5)* Radical notation is a representation of rational exponents.	How can students use properties of exponents to simplify products and quotients of radicals? How can students write general rules involving properties of exponents?	**IFL Sets of Related Lessons "Investigating Rational Exponents"**

		Rewrite expressions containing rational exponents into radical form. Rewrite expressions containing radical notation into exponential expressions containing rational exponents. Rational exponents are exponents that are fractions. Properties of integer exponents extend to properties of rational exponents. Properties of rational exponents are used to simplify and create equivalent forms of numerical expressions. Rational exponents can be written as radicals, and radicals can be written as rational exponents. **SPED Strategies:** Pre-teach vocabulary and provide verbal and pictorial descriptions to maximize understanding and interest. Review the rules of exponents with students and provide them with a reference document highlighting rules and examples. Model the thinking and processes involved in converting expressions involving radicals	How can students write and evaluate the nth root of a number? How do students use properties of rational exponents to simplify and create equivalent forms of numerical expressions? Why are rational exponents and radicals related to each other? Given an expression with a rational exponent, how do you write the equivalent radical expression? Certain properties govern operations with terms involving exponents. They include: • the multiplication property of exponents which states that $a^m \times a^n = a^{m+n}$ • when $a \neq 0$, the division property of exponents which states that $\frac{a^m}{a^n} = a^{m-n}$,	**Type II, III: Evaluating a Special Exponential Expression# 1** **Evaluating Exponential Expressions # 2** **Extending the Definitions of Exponents, Variation 2** **Rational or Irrational**

		and rational exponents. Provide students with opportunity to practice with peers and encourage them to verbalize and justify their thinking process.	• the power property of exponents which states that $(a^m)^n = a^{(m \times n)}$; and • when $b \neq 0$, the regrouping property of exponents which states that $(ab)^n = a^n \times b^n$ and $(\frac{a}{b})^n = \frac{a^n}{b^n}$. These properties apply to all real number exponents.	

New Jersey Student Learning Standard(s):

A.SSE.B.3: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression

A.SSE.B.3c: Use the properties of exponents to transform expressions for exponential functions. *For example the expression* 1.15^t *can be rewritten as* $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*

F.IF.C.8: Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function

F.IF.C.8b: Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, *and classify them as representing exponential growth or decay.*

Student Learning Objective 8: Use the properties of exponents to transform expressions for exponential functions, explain properties of the quantity revealed in the transformed expression or different properties of the function.

Modified Student Learning Objectives/Standards:

M.EE.A-SSE.3: Solve simple algebraic equations with one variable using multiplication and division.

MPs	Evidence Statement Key/ Clarifications	Skills, Strategies & Concepts	Essential Understandings/ Questions (Accountable Talk)	Tasks/Activities

MP 1 **MP 2** **MP 4** **MP 7**	**A-SSE.3c-2** • Tasks have a real-world context. • The equivalent form must reveal something about the real-world context. • Tasks require students to make the connection between the equivalent forms of the expression. **F-IF.8b**	Alternate, equivalent forms of an exponential expression containing rational exponents may reveal specific attributes of the function that it defines. Use properties of exponent transform/rewrite an exponential expression for an exponential function. Explain the properties of the quantity or the function. **SPED Strategies:** Pre-teach vocabulary and provide verbal and pictorial descriptions to maximize understanding and interest. Review the rules of exponents with students and provide them with a reference document highlighting rules and examples. Model the thinking and processes involved in rewriting exponential functions in equivalent forms. Explain and illustrate why rewriting exponential functions in equivalent forms helps solve problems more effectively.	What are some of the characteristics of exponential growth and exponential decay functions? How are properties of exponents used to transform expressions for exponential functions? Why would you want to transform an expression for an exponential function? How do different forms of the function help you to identify key features?	**Type II, III:** **Seeing Dots** **Forms of exponential expressions** **Analyzing Graphs** **Which Function Springboard Dive** **Additional Tasks:** **Carbon 14 Dating in Practice I** **Ice Cream** **Profit of a Company**

New Jersey Student Learning Standard(s):
F.LE.A.4: Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

Student Learning Objective 9: Express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

Modified Student Learning Objectives/Standards: N/A

MPs	Evidence Statement Key/ Clarifications	Skills, Strategies & Concepts	Essential Understandings/ Questions (Accountable Talk)	Tasks/Activities
MP 2 **MP 4**	**N/A**	Exponents and logarithms have an inverse relationship. Solutions to an exponential equation in one variable can be written as a logarithm. Transform an exponential model represented by $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e. Write the solution to $ab^{ct} = d$ as a logarithm. Use technology to evaluate logarithms having base 2, 10, or e. **SPED Strategies:** Pre-teach vocabulary and provide verbal and pictorial descriptions to	What is the natural base e? What are some of the characteristics of the graph of a logarithmic function? How can you use properties of exponents to derive properties of logarithms? How can you solve exponential and logarithmic equations? How do you evaluate a logarithm using technology? How do logarithms help you to solve exponential functions?	**Type II, III:** **Graphene** **Carbon 14 dating in practice I** **Accuracy of Carbon 14 Dating II** **Snail Invasion** **Newton's Law of Cooling** **Exponential Kiss** **Additional Tasks: Bacteria Populations** **Carbon 14**

		maximize understanding and interest. Explain and illustrate the relationship between exponents and logarithms side by side. Provide students with a reference sheet that illustrates the relationship between exponents and logarithms side by side to encourage confidence and independence.		**Snail Invasion**

Integrated Evidence Statements

A.Int.1: Solve equations that require seeing structure in expressions

- Tasks do not have a context.
- Equations simplify considerably after appropriate algebraic manipulations are performed. For example, $x^4-17x^2+16 = 0$, $2^{3x} = 7(2^{2x}) + 2^{2x}$, $x - \sqrt{x} = 3\sqrt{x}$
- Tasks should be course level appropriate.

F-BF.Int.2: Find inverse functions to solve contextual problems. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. For example, $f(x) = 2x^3$ or $f(x) = \frac{x+1}{x-1}$ for $x \neq 1$.

- For example, see http://illustrativemathematics.org/illustrations/234.
- As another example, given a function $C(L) = 750L^2$ for the cost C(L) of planting seeds in a square field of edge length L, write a function for the edge length L(C) of a square field that can be planted for a given amount of money C; graph the function, labeling the axes.
- This is an integrated evidence statement because it adds solving contextual problems to standard F-BF.4a.

F-Int.1-2: Given a verbal description of a polynomial, exponential, trigonometric, or logarithmic functional dependence, write an expression for the function and demonstrate various knowledge and skills articulated in the Functions category in relation to this function.

- Given a verbal description of a functional dependence, the student would be required to write an expression for the function and then, e.g., identify a natural domain for the function given the situation; use a graphing tool to graph several input-output pairs; select applicable features of the function, such as linear, increasing, decreasing, quadratic, periodic, nonlinear; and find an input value leading to a given output value.

HS-Int.3-3: Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in F-LE, A-CED.1, A-SSE.3, F-IF.B, F-IF.7★

- F-LE.A, Construct and compare linear, quadratic, and exponential models and solve problems, is the primary content and at least one of the other listed content elements will be involved in tasks as well.

HS.C.3.1: Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures about numbers or number systems. Content Scope: N-RN, N-CN

HS.C.3.2: Base explanations/reasoning on the properties of exponents. Content Scope: N-RN.A

HS.C.4.1: Derive and use a formula. Content Scope: A-SSE.4

Integrated Evidence Statements

HS.C.12.2: Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures about functions. Content scope: F-IF.8b.

HS.C.18.4: Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures about polynomials, rational expressions, or rational exponents. Content scope: N-RN, A-APR.(2, 3, 4, 6)

HS.C.CCR: Solve multi-step mathematical problems requiring extended chains of reasoning and drawing on a synthesis of the knowledge and skills articulated across: 7-RP.A.3, 7-NS.A.3, 7-EE.B.3, 8-EE.C.7B, 8-EE.C.8c, N-RN.A.2, A-SSE.A.1b, A-REI.A.1, A-REI.B.3, A-REI.B.4b, F-IF.A.2, F-IF.C.7a, F-IF.C.7e, G-SRT.B.5 and G-SRT.C.7.

- Tasks will draw on securely held content from previous grades and courses, including down to Grade 7, but that are at the Algebra II/Mathematics III level of rigor.
- Tasks will synthesize multiple aspects of the content listed in the evidence statement text, but need not be comprehensive.
- Tasks should address at least A-SSE.A.1b, A-REI.A.1, and F-IF.A.2 and either F-IF.C.7a or F-IF.C.7e (excluding trigonometric and logarithmic functions). Tasks should also draw upon additional content listed for grades 7 and 8 and from the remaining standards in the Evidence Statement Text.

HS.D.2-7: Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in A-CED, N-Q.2, A-SSE.3, A-REI.6, A-REI.7, A-REI.12, A-REI.11-2.

- A-CED is the primary content; other listed content elements may be involved in tasks as well.

HS.D.3-5: **Decisions from data: Identify relevant data in a data source, analyze it, and draw reasonable conclusions from it. Content scope: Knowledge and skills articulated in Algebra 2.**

- Tasks may result in an evaluation or recommendation. ii.) The purpose of tasks is not to provide a setting for the student to demonstrate breadth in data analysis skills (such as box-and-whisker plots and the like). Rather, the purpose is for the student to draw conclusions in a realistic setting using elementary techniques.

Integrated Evidence Statements

HS.D.CCR: Solve problems using modeling: Identify variables in a situation, select those that represent essential features, formulate a mathematical representation of the situation using those variables, analyze the representation and perform operations to obtain a result, interpret the result in terms of the original situation, validate the result by comparing it to the situation, and either improve the model or briefly report the conclusions. Content scope: Knowledge and skills articulated in the Standards as described in previous courses and grades, with a particular emphasis on 7- RP, 8 – EE, 8 – F, N-Q, A-CED, A-REI, F-BF, G-MG, Modeling, and S-ID

- Tasks will draw on securely held content from previous grades and courses, include down to Grade 7, but that are at the Algebra II/Mathematics III level of rigor.
- Task prompts describe a scenario using everyday language. Mathematical language such as "function," "equation," etc. is not used.
- Tasks require the student to make simplifying assumptions autonomously in order to formulate a mathematical model. For example, the student might make a simplifying assumption autonomously that every tree in a forest has the same trunk diameter, or that water temperature is a linear function of ocean depth.
- Tasks may require the student to create a quantity of interest in the situation being described.

Algebra II Vocabulary

Number and Quantity	Algebra	Functions		Statistics and Probability	
Complex number Conjugate Determinant Fundamental theorem of Algebra Identity matrix Imaginary number Initial point Moduli Parallelogram rule Polar form Quadratic equation Polynomial Rational exponent Real number Rectangular form Scalar multiplication of Matrices Terminal point Vector Velocity Zero matrix	Binomial Theorem Complete the square Exponential function Geometric series Logarithmic Function Maximum Minimum Pascal's Triangle Remainder Theorem	Absolute value function Asymptote Amplitude Arc Arithmetic sequence Constant function Cosine Decreasing intervals Domain End behavior Exponential decay Exponential function Exponential growth Fibonacci sequence Function notation Geometric sequence Increasing intervals Intercepts Invertible function	Logarithmic function Trigonometric function Midline Negative intervals Period Periodicity Positive intervals Radian measure Range Rate of change Recursive process Relative maximum Relative minimum Sine Step function Symmetries Tangent.	2-way frequency table Addition Rule Arithmetic sequence Box plot Causation Combinations Complements Conditional probability Conditional relative frequency Correlation Correlation coefficient Dot plot Experiment Fibonacci sequence Frequency table Geometric sequence Histogram	Independent Inter-quartile range Joint relative frequency Margin of error Marginal relative frequency Multiplication Rule Observational studies Outlier Permutations Recursive process Relative frequency Residuals Sample survey Simulation models Standard deviation Subsets Theoretical probability Unions

References & Suggested Instructional Websites

www.internet4classrooms.com

https://www.desmos.com/

http://nlvm.usu.edu/en/nav/index.html

https://www.georgiastandards.org/Georgia-Standards/Pages/Math-9-12.aspx

www.illustrativemathematics.org/

https://www.khanacademy.org/math/algebra-home/algebra2

http://www.mathplanet.com/education/algebra-2

https://www.ixl.com/math/algebra-2

http://www.mathsisfun.com/algebra/index-2.html

https://parcc.pearson.com/practice-tests/math/

https://www.illustrativemathematics.org/

http://map.mathshell.org/materials/lessons.php?gradeid=24

http://www.achieve.org/ccss-cte-classroom-tasks

http://www.nylearns.org/module/Standards/Tools/Browse?linkStandardId=0&standardId=97817

http://www.nciea.org/publications/Math_LPF_KH11.pdf

References & Suggested Instructional Websites

PARCC Model Content Frameworks. http://www.parcconline.org/sites/parcc/files/PARCCMCFMathematicsNovember2012V3_FINAL.pdf

PARCC Mathematics Evidence Tables. https://www.parcconline.org/assessment-blueprints-test-specs

Smarter Balanced Assessment Consortium. http://www.smarterbalanced.org/

Statistics Education Web (STEW). http://www.amstat.org/education/STEW/

The Data and Story Library (DASL). http://lib.stat.cmu.edu/DASL/

The High School Flip Book Common Core State Standards for Mathematics.
http://www.azed.gov/azcommoncore/files/2012/11/high-school-ccss-flip-book-usd-259-2012.pdf

Field Trip Ideas

Field Trip Ideas

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MUSEUM of MATHEMATICS: Mathematics illuminates the patterns that abound in our world. The National Museum of Mathematics strives to enhance public understanding and perception of mathematics. Its dynamic exhibits and programs stimulate inquiry, spark curiosity, and reveal the wonders of mathematics. The Museum's activities lead a broad and diverse audience to understand the evolving, creative, human, and aesthetic nature of mathematics.
www.momath.org

LIBERTY SCIENCE CENTER - An interactive science museum and learning center located in Liberty State Park. The center, which first opened in 1993 as New Jersey's first major state science museum, has science exhibits, the largest IMAX Dome theater in the United States, numerous educational resources, and the original *Hoberman sphere*.

http://lsc.org/plan-your-visit/